Challenge 245: A Curious Competition

Triangle chess is a game for three players, so any game involves three people.

Five students (A, B, C, D, E) are taking part in a triangle chess tournament.

The organisers want *each* pair of students to play together in *exactly* one game.

They make this plan for the tournament.

game 1: A, B, C

game 2: A, D, E

game 3: B, D, C

game 4: E, A, B

However, this is *not* satisfactory for two reasons

- the pairs A-B, A-E and B-C play together in two of the games

- the pair C-E do not play together in any game.

a Explain to the organisers why they cannot hope to have *each* pair of students play together in *exactly* one game.

b Show that the organisers cannot hope to have *each* pair of students play together in *exactly* one game even if there are six players.

c Provide the organisers with a satisfactory plan for the tournament if there are seven players.

d Explain why tournaments cannot be organised in this way with an even number of players, nor with a number of players which is one less than a multiple of 3.

e Identify the next number of players above seven for which a tournament **can** be organised in this way, and provide a plan for the tournament in this case.