UK Junior Mathematical Olympiad 2014

Organised by The United Kingdom Mathematics Trust

Thursday 12th June 2014

RULES AND GUIDELINES : READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING

1. Time allowed: 2 hours.
2. **The use of calculators, measuring instruments and squared paper is forbidden.**
3. All candidates must be in *School Year 8 or below* (England and Wales), *S2 or below* (Scotland), *School Year 9 or below* (Northern Ireland).
4. For questions in Section A *only the answer is required*. Enter each answer neatly in the relevant box on the Front Sheet. Do not hand in rough work. Write in blue or black pen or pencil.

   For questions in Section B you must give *full written solutions*, including clear mathematical explanations as to why your method is correct.

   Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left corner with the Front Sheet on top.

   Do not hand in rough work.
5. Questions A1-A10 are relatively short questions. Try to complete Section A within the first 30 minutes so as to allow well over an hour for Section B.
6. Questions B1-B6 are longer questions requiring *full written solutions*.
   This means that each answer must be accompanied by clear explanations and proofs.
   Work in rough first, then set out your final solution with clear explanations of each step.
7. These problems are meant to be challenging! Do not hurry. Try the earlier questions in each section first (they tend to be easier). Try to finish whole questions even if you are not able to do many. A good candidate will have done most of Section A and given solutions to at least two questions in Section B.
8. Answers must be FULLY SIMPLIFIED, and EXACT using symbols like π, fractions, or square roots if appropriate, but NOT decimal approximations.

DO NOT OPEN THE PAPER UNTIL INSTRUCTED BY THE INVIGILATOR TO DO SO!

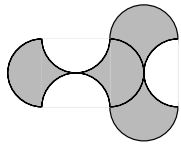
Section A

A1. What is the largest digit that appears in the answer to the calculation $(3 \times 37)^2$?

A2. What is the sum of all fractions of the form $\frac{N}{7}$, where N is a positive integer less than 7?

A3. The six angles of two different triangles are listed in decreasing order. The list starts 115°, 85°, 75° and 35°. What is the last angle in the list?

A4. The figure shows two shapes that fit together exactly. Each shape is formed by four semicircles of radius 1. What is the total shaded area?



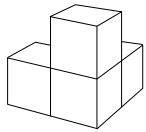
A5. The integer 113 is prime, and its 'reverse' 311 is also prime. How many two-digit primes are there between 10 and 99 which have the same property?

A6. A square of side length 1 is drawn. A larger square is drawn around it such that all parallel sides are a distance 1 apart. This process continues until the total perimeter of the squares drawn is 144.

What is the area of the largest square drawn?

A7. The time is 20:14. What is the smaller angle between the hour hand and the minute hand on an accurate analogue clock?

A8. Sam has four cubes all the same size: one blue, one red, one white and one yellow. She wants to glue the four cubes together to make the solid shape shown.



How many differently-coloured shapes can Sam make?

[Two shapes are considered to be the same if one can be picked up and turned around so that it looks identical to the other.]

A9. A rectangle is made by placing together three smaller rectangles P, Q and R, without gaps or overlaps. Rectangle P measures 3 cm × 8 cm and Q measures 2 cm × 5 cm.

How many possibilities are there for the measurements of R?

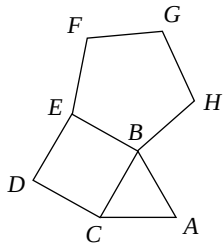
A10. My four pet monkeys and I harvested a large pile of peanuts. Monkey A woke in the night and ate half of them; then Monkey B woke and ate one third of what remained; then Monkey C woke and ate one quarter of the rest; finally Monkey D ate one fifth of the much diminished remaining pile. What fraction of the original harvest was left in the morning?

Section B

Your solutions to Section B will have a major effect on your JMO results. Concentrate on one or two questions first and then **write out full solutions** (not just brief 'answers').

B1. The figure shows an equilateral triangle *ABC*, a square *BCDE*, and a regular pentagon *BEFGH*.

What is the difference between the sizes of $\angle ADE$ and $\angle AHE$?



B2. I start at the square marked A and make a succession of moves to the square marked B. Each move may only be made downward or to the right. I take the sum of all the numbers in my path and add 5 for every black square I pass through.

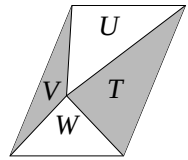
How many paths give a sum of 51?

A		12		10
	11		11	
10		10		15
	11		14	
10		13		B

B3. A point lying somewhere inside a parallelogram is joined to the four vertices, thus creating four triangles *T*, *U*, *V* and *W*, as shown.

Prove that

$$\text{area } T + \text{area } V = \text{area } U + \text{area } W.$$



B4. There are 20 sweets on the table. Two players take turns to eat as many sweets as they choose, but they must eat at least one, and never more than half of what remains. The loser is the player who has no valid move.

Is it possible for one of the two players to force the other to lose? If so, how?

B5. Find a fraction $\frac{m}{n}$, with m not equal to n, such that all of the fractions

$$\frac{m}{n}, \frac{m+1}{n+1}, \frac{m+2}{n+2}, \frac{m+3}{n+3}, \frac{m+4}{n+4}, \frac{m+5}{n+5}$$

can be simplified by cancelling.

B6. The sum of four different prime numbers is a prime number. The sum of some pair of the numbers is a prime number, as is the sum of some triple of the numbers. What is the smallest possible sum of the four prime numbers?